

1. Use Kolmogorov's Axioms and the properties of sets to show that, in general,

$$P(AB) \leq P(A) \leq P(A+B) \leq P(A)+P(B)$$

2. On a recent trip to the Western Economic Association Meetings in Las Vegas, Professor Zuehlke observes Arnold Zellner playing a slot machine, losing 99 consecutive times, and stomping off in disgust. Professor Zuehlke immediately races from the bar to play the machine. Based upon years of experience teaching introductory econometrics, he has concluded that he cannot lose since the probability of 100 consecutive losses is virtually zero. Comments?
3. A random variable,  $X$ , has density  $f(x) = \theta^{-1}[\exp(-x/\theta)]$  for  $x > 0$  and  $\theta > 0$ .
- Find the MGF of  $X$ .
  - Find  $E(X)$  from the MGF.
  - Calculate  $E(X)$  directly.
4. Given the following joint density function:

|     | X=1 | X=2 | X=3 |
|-----|-----|-----|-----|
| Y=4 | 0.2 | 0.1 | 0   |
| Y=5 | 0   | 0.3 | 0.2 |
| Y=6 | 0   | 0   | 0.2 |

- Find the marginal density of  $X$ .
  - Find the mean of  $X$ .
  - Find the conditional density of  $Y$  given  $X$  equals 2.
  - Find the conditional expectation of  $Y$  given  $X$  equals 2.
  - Are  $X$  and  $Y$  statistically independent?
5. Show that if  $X$  and  $Y$  are statistically independent, and  $\gamma(X)$  and  $\eta(Y)$  are arbitrary transformations, then  $E[\gamma(X)\eta(Y)] = E[\gamma(X)]E[\eta(Y)]$ .